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OPTIMUM MAINTENANCE POLICIES - A LITERATURE SURVEY

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ABSTRACT

Components subject to failure periodically undergo preventive maintenance in an attempt to minimize costs associated with failure and repair. However, maintenance itself incurs some cost and, therefore, the costs of maintenance and repair after failure must be balanced. This report reviews the approaches to optimum maintenance modelling presented in the extensive literature on the topic. The focus is on work carried out since the 1976 survey of maintenance models by W.P. Pierskalla and J.A. Voelker.

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To Mr. E.J. Kee
Director of Research

OPTIMUM MAINTENANCE POLICIES - A LITERATURE SURVEY

1. INTRODUCTION

One of the major sources of operating expenses for industry, and a utility corporation in particular, is related to equipment maintenance and repair. For Ontario Hydro, these expenses run into tens of millions of dollars per year. Preventive maintenance on equipment is carried out in the expectation that, as a consequence, the mean time between failures will increase, thus costs associated with failures and repair will reduce. However, maintenance itself incurs some cost and usually requires that the equipment under maintenance be taken out of service. A maintenance decision thus involves a compromise between 'too little' and 'too much' maintenance.

A methodical approach to préventive maintenance planning and scheduling, if it results in saving even a small portion of the total expenses, should be worth its while. Several aspects of the problem have been studied in some detail in the literature and policies which are optimal in a specified sense have been described. However, in practice, maintenance decisions are often taken on a judgemental basis or governed by traditions that may no longer be valid.

While the complexity of the maintenance problem makes it difficult to model and solve, recent financial constraints on capital funding have resulted in renewed interest in maintenance management programs. This report summarizes several optimum maintenance policies studied in the literature and explores their possible relevance to Ontario Hydro's operations.

2. THE CONCEPT OF OPTIMUM MAINTENANCE

2.1 Fundamentals

In the context of this study, the term 'maintenance' includes several diverse but related activities such as repair, overhaul, replacement, inspection, etc. The term 'optimum maintenance policy' will be used to mean a policy chosen to obtain the best values of a given objective function. The details involved can

be best appreciated by considering the case of an equipment, on which major maintenance overhauls are performed periodically. In between overhauls, it is subject to loss of efficiency and/or breakdown. One would like to determine the optimum interval between overhauls to minimize the total cost. Figure 1 shows the two kinds of conflicting costs involved:

C_1 = operating costs, including costs of repair in between overhauls and costs due to loss of efficiency,

C_2 = maintenance costs, including loss of production during maintenance.

One assumes that increasing the maintenance frequency reduces repair frequency and associated costs. Thus, C_1 increases as the maintenance interval increases, and C_2 naturally decreases. If C_1 and C_2 can be determined as functions of the maintenance interval, the optimum interval to minimize the total cost $C = C_1 + C_2$ can be obtained as shown.

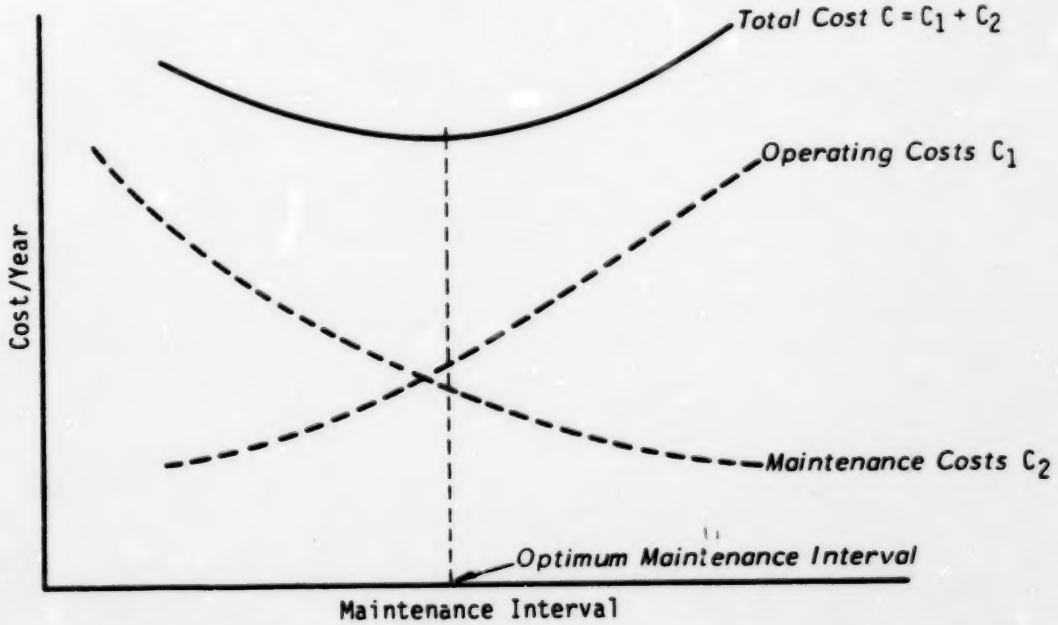

FIGURE 1

OPERATING AND MAINTENANCE COSTS VERSUS MAINTENANCE INTERVAL

The main ingredients of a systematic approach to maintenance management can now be stated:

1. Identifying the decision variables.

2. Choosing a performance criterion.

3. Developing a model.

Each element is discussed briefly below.

2.2 Decision Variables

As mentioned before, a maintenance decision could involve several variables. Some of these studied in the literature are the following.

- Component Replacement Frequency

  The decision involves the question, how often should a component be replaced. The optimum solution seeks a compromise between the cost of new components and the costs associated with the use of less efficient and/or less reliable components.

- Preventive Maintenance Frequency

  As outlined in the example above, the question is whether to carry out preventive maintenance (if so, how often?) or wait till a breakdown occurs. In the solution, operating (which includes repair) and maintenance costs must be balanced.

- Inspection Times

  Often, the state of deterioration of an equipment can be detected by inspection (at a cost). A short inspection interval will increase reliability whereas a long interval lowers inspection costs.

- Ordering and Providing Spares

  Here, the decision involves the number of spares to be ordered and the times at which they should be ordered.

Clearly, decisions can be made at different hierarchical levels, eg, at the component, equipment or site levels, with possible interactions between them. Decisions at the system level can include a variety of problems, such as:

a) optimum maintenance crew size,

b) allotment of maintenance budget,

c) scheduling of maintenance of several similar equipments.

The effects of a given maintenance policy cannot really be separated from operating decisions. Thus, in a sense, most maintenance decisions will be made at the system level when one seeks a policy which is 'best' for the organization as a whole. Nevertheless, one can often consider a problem ignoring the interactions either because the interactions are small or they are clearly known and thus their effects can be included as problem constraints.

2.3 Choosing a Performance Criterion

The second task in maintenance optimization is to decide upon an index which is a measure of the maintenance performance. Not surprisingly, most indices are related to the reliability of the equipment and the costs involved. Some commonly chosen criteria are:

- maximizing availability,
- minimizing total cost,
- maximizing profit,
- maximizing efficiency.

Depending upon the time period involved, the costs can include effects of discounting, interest rates, etc. Often, the problem specification involves one or more constraints. In some cases, it may not be possible to specify quantitatively the required criterion. The decision-maker should then be presented with the results obtained in several alternatives with different criteria (eg, efficiency, cost, availability, etc) and the final choice is based upon his judgement.

2.4 Development of a Model

This step essentially involves the determination of the relationship between the performance criterion and the decision variables. In the example studied above, one needs to obtain as a function of the maintenance interval,

a) the repair costs C_1 (including costs of shutdown after failure and performance degradation)

b) the maintenance costs C_2 (including costs of shutdown to perform maintenance).

The costs involve the effect of the maintenance interval on the performance of the equipment and cannot usually be determined in a straightforward manner. Mathematical models used to determine the needed relationships can be roughly divided into two classes: deterministic and probabilistic.

- In a deterministic model, the effect of each maintenance action is known with certainty. In the above example, if M is the maintenance interval, one could model C_1 and C_2 as

$$C_1 = a_0 + a_1 M + a_2 M^2$$

$$C_2 = b_0 / M$$

If historical data are available, one obtains a_0, a_1, a_2 and b_0 by simple curve fitting.

- A probabilistic model recognizes the fact that the time to failure is a random variable and characterizes it by its probability distribution. The probability distribution is estimated from historical data and the costs are replaced by their expected values.

2.5 Structure of the Literature Survey

The area of maintenance modelling has been surveyed thoroughly upto 1976 in two studies: Barlow and Proschan (1965) and Pierskalla and Voelker (1976). In the current review, the focus is on work appearing since 1976. Some of the earlier results are also summarized.

For the purposes of the current work, most of the study systems are classified into two categories: Class I (or simple systems) and Class II (or complex systems). Class I systems are defined to be those which (not considering maintenance) can be in only two states - an operating state and a failed state. Class II systems on the other hand, are capable of assuming several operating states with varying degrees of deterioration. These two categories are studied in Sections 3 and 4 respectively. Models not included as Class I or Class II are discussed in Section 5. Section 6 discusses the two unit standby system and Section 7 reviews maintenance policies in the industry.

A different classification system is used in a recent survey by Sherif and Smith (1981).

3. CLASS I OR SIMPLE SYSTEM MODELS

As mentioned earlier, systems which, when not in maintenance, can be in only one of two states: operating and failed, are in this report termed 'Class I' systems. Most maintenance models for such systems can be divided into two groups: discrete and continuous. In discrete (or classical) models, the maintenance activity is limited to actions at discrete time intervals. In continuous (or control theory) models, the maintenance activity may occur "continuously" as the needs of the system require. Typically, the maintenance decision involves the rate of maintenance expenditure as a function of time. The two groups are considered in Sections 3.1 and 3.2, respectively. Section 3.3 discusses models which cannot be strictly classified into either of the above two categories.

3.1 Discrete (or Classical) Models

The earlier models are described in detail in Barlow and Proschan (1965). In this reference, the following policies are identified for a single unit with a known distribution for the time to failure:

(a) Replace at the time of failure or at time T after installation (age replacement).

(b) Replace at times T, 2T, 3T, ... and at failure (block replacement).

(c) Replace at times T, 2T, 3T, ... with minimal repair at failure.

In the literature (a) is called Policy I and (c), Policy II. In the above policies, one could carry out maintenance on the unit and make it 'as good as new', instead of replacing it. After a minimal repair, the unit is in the same condition as before the failure, that is, the unit is 'as bad as old'. The costs of repair and replacement are known.

Barlow and Proschan (1965) gave conditions under which an optimum T exists to minimize the expected cost per unit time and obtained expressions to compute the optimum value of T for the various policies. The expected costs (C_1 and C_2 in Figure 1) depend on the failure distribution and therefore several specific distributions were investigated.

Work extending the above model is subdivided in this study as follows: Section 3.1.1 outlines generalizations including

modified policies, changes in objective function etc. Solutions for specific failure distributions and comparisons of various policies are reviewed in Section 3.1.2. Imperfect preventive maintenance is considered in Section 3.1.3 and finally, inspection in Section 3.1.4.

3.1.1 Modification of Barlow and Proschan's Models

Several modifications have been described in the survey paper by Pierskalla and Voelker (1976). These included age dependent costs, discounting, and different operating policies. Work appearing since the above survey is outlined below.

Several authors have studied the use of modified objective functions. Osaki and Nakagawa (1975) considered the effect of discounting. They also summarized some earlier results. Kay (1976) maximized the incremental revenue and obtained an approximate solution when the time to failure has a Weibull distribution. Osaki and Yamada (1976) took into account the delivery times of the replacement units. Mine and Nakagawa (1977) maximized the interval reliability when the interval is (a) constant and (b) distributed exponentially.

Nakagawa (1977) discussed several criteria including earning rate, availability, interval reliability etc. He also summarized some earlier results for 1-unit and 2-unit standby systems. Kaio and Osaki (1978) took into account the salvage cost for a non-failed unit. Handlarski (1980) optimized the profit (reliability cost less the maintenance cost) for the age replacement and the minimal repair policies. He also considered the case where the quality of breakdown repair is inferior to the quality of maintenance repair. Nummelin (1980) considered the case where the failure cost and the cost of replacement before failure both depend on the age of the unit. Boland and Proschan (1982) studied a minimal repair policy under the assumption that the cost of minimal repair increases with each failure. They obtained expressions for the optimal period T for both finite and infinte time horizons.

Models where the classical operation-failure-maintenance cycle is retained, but the policies are more complex are discussed next. Muth (1977) considered a modification of the minimal repair policy of Barlow and Proschan. In his policy, minimal repair is performed till a time T and the unit is replaced at the next failure. He used a non-standard failure distribution for analytical tractability. Park (1979) studied a policy where the unit is replaced at the kth failure, the previous (k-1) failures being corrected by minimal repair. He showed that it

is more efficient than the minimal repair policy when the failure distribution is Weibull. Tango (1979) proposed a modified block replacement policy to account for the fact that, at time kT, the item might be almost new. In this policy, if a unit fails before kT, it is replaced not by new units but by used (less reliable) units.

Kapur, Kapoor and Kapil (1980) studied a joint policy of preventive maintenance and repair limit maintenance simultaneously. Nakagawa (1980) considered a unit which has a constant failure rate upto time T_0 (period I) and increasing failure rate after T_0 (period II). He studied three different policies for such a unit: (a) replace at failure or at $T_0 + T$, (b) replace at $T_0 + T$ with minimal repair till $T_0 + T$, and (c) minimal repair in period I and failure replacement in Period II. Adachi and Kodama (1980, 1980a) studied a policy similar to that of Park (1979) where a preventive maintenance is performed after a total operating time of T and a corrective maintenance is done at the kth failure, the previous (k-1) failures being corrected by minimal repair. Nguyen and Murthy (1980) extended the repair limit policy of Nakagawa and Osaki (1974a) by considering a random repair limit time. In this policy, if the repair is not completed within a time T, the unit is replaced by a new unit. They showed that when the expected cost per unit is minimized, the original results are optimal for both deterministic and random T. Nakagawa (1981) proposed modified policies for the case where failure occurs just before a planned replacement time. Kaio and Osaki (1981a) considered a repair limit policy with a cost constraint, that is, the unit is repaired if the anticipated repair cost is less than a limit cost; if not, it is replaced. Nguyen and Murthy (1981) studied the case where the failure rate is changed after each repair and obtained equations to compute the optimum planned replacement times T_i after the ith repair.

Nakagawa (1982) considered a modified block replacement policy where if a failure occurs in the interval T_0 to T, the unit remains failed until time T. He obtained expressions to compute the optimum T_0 and T to minimize the cost per unit time. Beichelt (1982) proposed a modification of the repair limit policy where instead of the estimated repair cost, the actual average repair cost to date is used.

Several other modifications which do not fall strictly into either of the above two subcategories have also been reported. Nakagawa (1976) studied the case where the main unit is repairable but the spares are not. Nakagawa and Osaki (1976) converted several earlier results to systems operating at discrete

times. Wolff (1979) studied a multicomponent equipment with minimal repair for the failed components. Beichelt and Fischer (1980) studied a failure model with two types of failure (with different associated repair times and costs). Beichelt (1981) extended the above result to the block replacement policy. Murthy and Maxwell (1981) studied the case where two types of replacement parts with different failure rates are available. This difference is known only after testing and they obtained conditions under which it is advantageous to test. Kowada and Bando (1982) considered a unit with two types of repair. Repair I is made at each kth failure ($k \neq nN$) and Repair II at each nNth failure. After Repair I, the age of the unit becomes a fraction of the total operating time before failure, and after Repair II, the unit is as good as new.

3.1.2 Comparisons and Some Specific Solutions

As seen in the previous section, a number of policies have been proposed since the original methods of Barlow and Proschan. This section discusses papers dealing with the implementation of the policies including comparisons and solutions for specific lifetime distributions.

Glasser (1967) obtained graphical solutions to the age replacement problem for the Weibull, the gamma and the truncated normal distributions. Berg (1976) showed that, under certain conditions, the age replacement policy is the optimum decision rule among several reasonable policies. Cheng (1977) studied the optimal replacement rule for devices with lognormal failure distribution. Nakagawa and Yasui (1978) suggested a method for approximate calculation of the block replacement for a Weibull distribution. Berg and Epstein (1978) compared the age, block and failure (ie, no maintenance) policies for a two-stage-Erlang distribution.

Beichelt and Fischer (1979) showed that solution for some general policies can be reduced to a form similar to that for age replacement. Hastings and Jardine (1979) discussed three methods of obtaining solutions for the age and block replacement policies from field data using: (a) probability distribution fitting (b) probability paper and (c) a computer program. Nakagawa (1979) studied the problem of finding the suboptimum replacement interval when the unit could not be replaced exactly at the optimal replacement time and gave a numerical example using the Weibull distribution. Bergman (1980) obtained a graphical solution for the above problem. Tadikamalla (1980) studied the age replacement policy and tabulated the optimum solution for various values of the parameters of a Weibull distribution. He also performed a sensitivity analysis for the model.

Nakagawa and Yasui (1981) obtained two asymptotic formulae for the lower and upper bounds for the age replacement policy for a Weibull distribution. Phelps (1981) compared three policies for a Weibull distribution: (a) Barlow and Prochan's policy of minimal repair upto T and replacing at T, (b) Park's (1979) policy of minimal repair for k-1 failures and replacing at the kth failure and (c) Muth's (1977) policy of minimal repair upto T and replacing at the next failure. He showed that policy (c) is the best and policy (b) is the second best. However, because policy (c) is very difficult to solve, he suggested using either policy (b) or policy (c) with the T computed using policy (a). Borgefors and Hjorth (1981) studied the effect of errors in estimating the failure distribution parameters when the sample size is small. Bergman and Klefsjo (1982) studied a graphical solution to the age replacement problem using the scaled TTT (total time on test) transform.

3.1.3 Imperfect Preventive Maintenance

Maintenance is said to be imperfect when the 'as good as new' assumption is no longer valid for the state after maintenance. Chan and Downs (1978) considered the case where preventive maintenance causes immediate failure with a probability p. Nakagawa (1979a) assumed that the unit after maintenance has the same failure rate as before with a probability p and is as good as new with a probability (1-p), and obtained expressions for the optimum T under the age replacement and minimal repair policies. Nakagawa (1979b) considered the case where improvement after a preventive maintenance is a function of the resources devoted. He obtained an expression for the optimum resources to minimize the expected cost per unit time under the minimal repair policy. Ohashi and Nishida (1980) proposed a general model including minimal repair where after each preventive maintenance, the unit is assumed to enter a different state. Murthy and Nguygen (1981) considered a model similar to that of Chan and Downs (1978) where preventive maintenance causes immediate failure with a probability p and showed that maintenance should not be performed for p greater than a certain value p'. Nguyen and Murthy (1981a) proposed a model where the failure distribution after preventive maintenance is different from that after a corrective maintenance. In the policy considered, the operating time before the next preventive maintenance depends on whether the previous maintenance was a preventive or a corrective one.

3.1.4 Models with Inspection

The maintenance models considered so far assume that the state of the system (whether it is operating or has failed), is always

known with certainty. This section studies cases where a system condition where repair is required can be identified only through inspection. For example, a malfunction in a machine may be determined only by periodic inspections of the quality of its product. There is a cost C' for each inspection and a damage cost C" per unit time from the time of failure until the detection of failure. The problem is to determine the optimal number of checks and the optimal times at which they should be performed to minimize the total cost.

Barlow and Proschan (1965) analyzed this problem and obtained expressions for the optimal solution to minimize the expected cost per unit time. Keller (1974) obtained simpler approximate solutions to the problem for the case when the checks are very frequent (ie, inspection costs are low). Keller (1982) obtained a better approximation using an asymptotic solution to Barlow and Proschan's equations.

Sherwin (1979) proposed a model for the case where failures are self announcing but inspection can detect signs of failure and obtained equations for the optimum inspection schedule. The model includes the possibility of erroneous inspection assessment. In the model proposed by Menipaz (1979), discounted costs and changes in maintenance costs with time are included. He also discussed a dynamic programming solution to the problem.

Nakagawa (1980a) extended Barlow and Proschan's (1965) model to the case where maintenance is not perfect, but the age of the unit becomes a fraction of the age t before maintenance. Yamada and Osaki (1980) included a delay between a request for inspection and the actual inspection.

3.2 Continuous or Control Theory Models

In these models, the maintenance activity is modelled by a continuous function of time and the object is to optimize the maintenance expenditure rate as a function of time. Such a model will be more suited to the system level rather than the component level. Thomson (1968) presented a simple model where the salvage value of the equipment, S(t), is the solution of the differential equation

$$dS(t)/dt = f(t)\, m(t) - d(t)$$

where m(t) is the maintenance expenditure rate, f(t) is a maintenance effectiveness function and d(t) is the rate of deterioration. The income function Q(t) is given by

$$Q(t) = p(t)\, S(t) - m(t)$$

where p(t) is the production rate. The optimum policy m(t) and the optimum life T maximize the present worth V(T) given by

$$V(T) = S(T)e^{-rT} + \int_0^T Q(t)\, e^{-rt}\, dt$$

where r is the interest rate. Such a formulation resembles the classical optimal control problem. Thomson showed that the optimum policy is a bang-bang type, ie, is piecewise constant. Extensions by Arora and Lele (1970) to include technological progress, by Kamien and Schwartz (1971) to take into account the increasing probability of machine failure with age, as well as a few others are outlined in the survey paper by Pierskalla and Voelker (1976).

Machine failure probability was also considered by Alam and Sarma (1974) who showed that when it is independent of maintenance, the optimal policy is still the bang-bang type. Sarma and Alam (1975, 1976) took into account the possibilities of repair and the waiting line formed at the repair facility. They obtained expressions for the optimal control policy and suggested methods to calculate the optimum crew size. Alam and Sarma (1977) studied the problem with one repairman.

Ahmed and Schenk (1978) defined the objective function in terms of costs of repair and downtime. They also considered variable as well as fixed maintenance policies. Tapiero and Venezia (1979) proposed a stochastic deterioration model with n Markovian states for a machine. (Note that while this is a control theory model, it is also a typical Class II system model discussed in Section 4). They also studied risk aversion in the decision maker by introducing corrections to the objective function which depend on the variance of the profit. Scott and Jefferson (1979) proposed bilinear control model which includes a variety of maintenance policies. Khandelwal et al (1979a) specified starting and ending times for the preventive maintenance and obtained optimum values for these times. Virtanen (1982) presented a different method of solution to the problem of machines subject to random failure.

4. CLASS II OR COMPLEX SYSTEM MODELS

In this study, Class II systems are defined as those which, when not failed or in maintenance, can be in one of a number of possible operating states (as opposed to Class I systems which are assumed to be in one of two states: operating or failed). Each state is characterized by the amount of deterioration and the deterioration is normally described by a Markov model.

In these models, the state of the unit is determined by an inspection at discrete points in time and a maintenance decision is taken depending on the degree of deterioration observed. The problem was first formulated using Markov decision theory by Derman (1962), Flehinger (1962) and Klein (1962). The unit is inspected at times n = 0, 1, 2... and is found to be in one of the states [0, 1...L]. 0 is the state when the unit is new and L is the failed state. After inspection, a decision is made either to replace or do nothing.

If X_n is the state at time n, $\{X_n\}$ forms a Markov chain described by the transition probability matrix $[p_{ij}]$ which is a function of the nature of the unit. If the unit is replaced only at failure, one also has

$$p_{i0} = 0 \qquad i < L$$

$$p_{L0} = 1$$

$$p_{iL}^{(t)} > 0 \qquad \text{for some } t \text{ and each } i < L$$

Maintenance action returns the unit to state 0 from an intermediate state and results in a modified Markov chain. It is assumed that the cost of replacing an unfailed unit is less than the cost of replacing a failed unit. The maintenance decision involves partitioning the state space into two categories: states at which the unit is replaced and states at which it is not replaced.

If the underlying distribution has a non-decreasing hazard rate, the optimal replacement rule to minimize the expected cost per unit time is a "control limit rule": Replace if the observed state is $\geq k$. The state k which is determined from the transition probability matrix is called the control limit.

Kolesar (1966) extended the above result by considering an occupancy cost for each state. Kao (1973) modelled the deterioration by a semi-Markov process and showed that the optimal policy is still given by a control limit rule. Several other extensions including treatment of random costs, noisy observations etc, are described by Pierskalla and Voelker (1976). Work appearing since the above review is listed below.

Mine and Kawai (1975) and Luss (1976, 1977) extended the basic result by not predetermining the inspection interval. Mine and Kawai (1975) showed that the control limit rule still holds and the optimum inspection interval becomes shorter as degradation

progresses. Luss (1976) considered a model with inspection costs, state occupancy costs, different costs for preventive and corrective maintenance, etc, and obtained optimum inspection and preventive schedules. Luss (1977) included the case where the unit is inoperative during inspection.

In most models, there are only two decision alternatives at each period: to do nothing or to replace. In Rosenfield's (1976) model, the optimal policy at each decision time chooses between three options: no action, inspection or replacement. Chikte and Kozin (1977) and Anderson (1981) studied models where various levels of preventive maintenance are available and the decision involves replacement, preventive maintenance or no action. In the optimal strategy, there is a control limit rule for replacement and the level of preventive maintenance is a nonincreasing function of the deterioration. Bobos and Protonotarios (1978) considered a model where the decision-maker may take at each stage one of several maintenance actions classified according to their expected effectiveness.

White (1979) studied the case where the observations on the system are noise corrupted. He showed that the optimal control rule type strategy needs a condition on the quality of the data as well. The model included the case of a completely unobserved system. Khandelwal et al (1979) considered a model for a production machine where maintenance cost increases linearly with the state of deterioration. A bisection algorithm to obtain the optimal solution to the basic problem was described by Thomas (1982).

5. OTHER MODELS

This section outlines some recent work relating to multi-component equipment etc, which cannot be strictly classified into the previous groups.

A critical problem in power system operation is the scheduling of the maintenance of generating units. Yamayee (1982) described the problem and provided a comprehensive survey of the literature.

Derman et al (1978), Smith (1978) and Derman and Smith (1979) studied the problems of a system with a given operating horizon where a certain component must be replaced whenever it fails (eg, the battery in a car). A number of spare categories with different costs and life distribution are available. They obtained expressions for the optimal category for replacement at each failure. Thomas (1982) described a bisection algorithm to obtain the optimal solution to the problem. Assaf and Levinson

(1982) extended the results to the case where the life distributions are arbitrary instead of the exponential distribution assumed in the earlier work.

Crabill (1974) studied a system with a finite number of machines and a repair facility which can operate at several rates. He obtained an expression for the optimal repair rate. Winston (1977) studied a similar system and discussed both continuous and discrete time maintenance. Smith (1978a) analyzed the problem where a single repairman maintains a series system whose components have exponential failure and repair distributions. He optimized the order of repair in order to maximize the work time. Albright (1980) studied the problem with m+n identical machines where as many as n can be operating and the rest are spares. The machines have exponential failure and repair distributions and there are S repairmen. He obtained expressions for the optimum values of S and the rates to minimize the total discounted cost.

D'Aversa and Shapiro (1978) studied a deterministic machine replacement model including tax depreciation, opportunity costs etc. The increasing probability of machine failure was represented by maintenance costs that increase with time. They obtained the solution using dynamic programming.

A maintenance model where damage to the unit is not intrinsic to it, but is due to external shocks is termed a shock model. The shocks are generally assumed to occur according to a Poisson process and the random damage due to each shock accumulates additively. Some results were reported in the survey paper by Pierskalla and Voelker (1976). Lu and Saeks (1979) studied the problem of estimating the lifetime of a unit in a shock environment from measurements at periodic maintenance intervals and used it to determine the optimal replacement time. Chikte and Deshmukh (1981) considered such a unit under continuous preventive maintenance and obtained the optimum maintenance and replacement rules.

Vergin and Scriabin (1977) used a dynamic programming model to study the problem of maintenance scheduling for multicomponent equipment. They compared the results with commonly used policies and found that the proposed model results in considerable improvements. Inagaki et al (1978) treated the problems of optimal preventive maintenance and optimal reliability allocation simultaneously and showed that it can be solved using a nonlinear mixed integer programming model. They also suggested a method of obtaining an approximate solution. Haurie and L'Ecuyer (1982) studied a multicomponent system with a number of identical elements. When one element fails, it is replaced

immediately and any number of working elements can be replaced preventively. They obtained a dynamic programming equation for the optimal strategy to minimize the total cost.

Different performance criteria (eg, minimum cost rate or maximum availability) normally yield different solutions. Hwang et al (1979) discussed four methods of choosing from among these results and the implications.

6. A TWO-UNIT STANDBY SYSTEM

Several authors have studied the applications of the above mentioned policies to a two-unit standby system where one of two identical units is in operation and the other is in standby status. The standby takes over when the operating unit fails or is under preventive maintenance.

Osaki (1972) modelled the system as a Markov Renewal Process and discussed the optimum maintenance schedule (under the age replacement policy) to maximize the expected time to first failure. Nakagawa and Osaki (1974) derived expressions for the optimum maintenance interval to maximize the steady-state availability. Nakagawa (1977) studied a priority standby system. He also obtained expressions to minimize the cost per unit time. Beicelt and Fischer (1979) gave a method for obtaining the approximate solution to maximize the expected lifetime.

Kumar and Kapoor (1981) obtained expressions to maximize the profit when the units have exponential lifetime distributions and arbitrary repair distributions. The model included effects of discounting, changing earning rate, etc.

Osaki and Okumoto (1977) studied a system with two phase repairs. Sinha and Kapil (1980) and Kapil and Sinha (1981) also studied a similar system and obtained expressions to maximize the availability.

Ohashi et al (1978) proposed the minimal repair policy for the two-unit system, where the standby operates only during maintenance of the main unit. They discussed the optimum schedule to maximize the mean time to first failure. Sugasawa and Hatate (1979) studied a lightly loaded two-unit system and again maximized the mean time to first system failure.

Mine and Kawai (1974) modelled each unit as a Class II system, ie, with one good, one failed and several degraded states. They discussed the maximization of the expected profit rate but no solution methods was given. Kumar et al (1981) gave a computer algorithm for the solution of the above problem. Kawai (1981)

showed that when the two units have identical Markovian degradation properties, the optimum maintenance policy to maximize the system availability is a control limit rule.

7. MAINTENANCE POLICIES IN PRACTICE

Most maintenance practices reported in the literature are system oriented rather than equipment oriented and concentrate on the interrelationships between the several organizational areas, namely, design, operation, maintenance, accounting, etc. The book edited by Jardine (1970) contains several case-study papers including maintenance crew size studies, optimum overhaul policies and maintenance management practices. This section reviews some recent papers describing existing maintenance management practices as well as some theoretical studies directed towards specific plant applications. Section 7.1 deals with general applications and Section 7.2 with applications in the electric power industry.

7.1 General Applications

Several authors have pointed out the need for a systematic approach to the maintenance management problem and the need to recognize interactions between various organizational functions. These include Baldin (1976, 1979, 1981), Yardley (1977), Bannister et al (1979), Baker (1980), and Fast (1982). The system normally involves automated preventive maintenance programs which use digital computers for information storage and retrieval leading to better control and flexibility of the maintenance functions.

Basker et al (1977) reported the successful application of Monte Carlo simulation methods to determine the optimum maintenance crew size for an engineering firm. Ramalingam and Watson (1977) obtained the optimal replacement schedule for metal cutting tools using a probabilistic model. They used a Weibull distribution to model the tool life. Weiss (1978) obtained optimal times for inspection and minor repairs in a chemical plant to minimize the cost and maximize the reliability at the plant level and discussed the need for coordination between maintenance and production departments. Lake and Muhlemann (1979) studied optimal replacement policies for wrapping machines in a biscuit factory using a deterministic model which takes into account resale value, depreciation etc. Sherwin and Lees (1981) reported the use of Weibull failure rate model to obtain the optimum maintenance policies for a chemical plant and a hospital autoclave system using Glasser's (1967) graphical solution. The discussion included other possible distributions as well as data

collection constraints. Richardson and McCarter (1981) discussed a computer based preventive maintenance system used in coal mines and construction projects and being adapted for power plant use. The system is primarily used to monitor each equipment and attend to those whose maintenance/performance ratio falls outside preset limits.

7.2 Power Industry Applications

O'Neill and Tupper (1975) examined common approaches to power plant maintenance. They evaluated various inspection policies such as single element inspection vs complete unit inspections and contract inspections vs in-house inspections. Brown and Cadick (1979) discussed the preventive maintenance program of a distribution substation and derived optimum inspection intervals for the power transformers, relays, circuit breakers etc, to minimize the total costs. Gatti (1979) reported obtaining optimum test intervals for relays using a fairly elaborate data collection system of the Philadelphia Electric Co. He discussed statistical analysis but fell short of using any mathematical models. Sahu (1980) reported a study on the optimum number of spare transformers on the American Electrical Power System.

Gheorghe (1979) reported the use of Markovian decision model for detection and optimum maintenance of fatigue cracks in a Light Water Reactor pressure vessel. Chavez and Perla (1980) described a maintenance management system developed for a nuclear power plant. The computerized system features automatic prioritizing, easy access to the complete failure data of the components, etc.

Sheliga (1981) used a deterministic model to obtain the optimum maintenance interval for an electric utility control and monitoring equipment. The number of failures per unit per year were assumed to be proportional to the square of the maintenance interval. Rifkin, Argue and Rogala (1981) described a computerized power generating station maintenance program to optimize time, manpower etc, and reported a saving of one million in the first year. Again, the primary function of the system is one of monitoring. Similar automated maintenance management systems for power plants were also reported by Bykov et al (1976) and Richardson (1980).

8. CONCLUSIONS

This report has presented a brief review of the techniques of optimum maintenance modelling and a survey of the relevant literature. The survey has not been a critical one and the

emphasis has been on providing a list of the topics that have been studied.

Two broad conclusions can be drawn:

(1) For a single unit or component subject to failure, several maintenance aspects (including inspection, imperfect maintenance, etc.) under a variety of policies have been studied thoroughly in the literature. However, very few applications are available. A possible reason is the lack of adequate input data.

(2) At the system or plant level, where decisions entail different levels and several components, there are not many theoretical studies available perhaps because of the complexities involved in modelling the interactions. Nevertheless, several industries have recognized the need for a computerized management system at the plant level. These systems primarily perform three functions:

    (a) program routine maintenance functions,

    (b) monitor them through the various stages,

    (c) record and assess the results.

    They also provide data for planning complex maintenance activities.

There is no doubt that a systematic approach to maintenance management will yield useful results. Specific recommendations toward such an approach are as follows:

(1) A thorough investigation of the maintenance practices within Ontario Hydro and the perceived needs for a maintenance management system should be performed. This will identify areas where such policies are already established and where improvements are possible.

(2) In areas where optimal maintenance policies are needed, the first priority should be to establish performance indices to measure the effectiveness of the policy. This should be done at a "convenient" hierarchical level and will be based on engineering judgement.

(3) The next stage will be the development of techniques to obtain the optimum policies. As listed earlier, this

includes identifying the critical variables and choosing a mathematical model to obtain the relationship between them. It should be possible to model most situations using one of the techniques discussed (tailored, if necessary, to the particular situation).

(4) There do exist computerized data collection schemes in Ontario Hydro. The critical aspect of the data analysis part will be the determination of the failure distribution in the presence of any maintenance action.

(5) A digital computer program to obtain the optimal solutions for any lifetime distributions for a variety of policies should be developed. Currently, most methods use the Weibull distribution because of its analytical tractability.

The following are a few theoretically oriented studies which are logical extensions of the above work:

(1) A sensitivity analysis of the optimum policies should prove very useful. For example, if the optimum policy requires a maintenance interval of twelve days, one would like to know the effects if the more acceptable intervals of ten or fifteen days are selected.

(2) The approximate Markov model, discussed at a recent meeting of the Reliability unit, where several states identify the gradual degradation of the unit and the inspection/maintenance activity is also modelled by a Markov state, should be investigated.

(3) The optimal policies depend on the exact failure distribution of the unit but in practice, scheduled maintenance at fixed intervals is performed on most units. Thus, the collected data are truncated. A method should be developed for the estimation of failure distributions in such cases.

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